Linear Network Optimization

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PREFACE

Linear network optimization problems such as shortest path, assignment, max-flow, transportation, and transhipment, are undoubtedly the most common optimization problems in practice. Extremely large problems of this type, involving thousands and even millions of variables, can now be solved routinely, thanks to recent algorithmic and technological advances. On the theoretical side, despite their relative simplicity, linear network problems embody a rich structure with both a continuous and a combinatorial character. Significantly, network ideas have been the starting point for important developments in linear and nonlinear programming, as well as combinatorial optimization.

Up to the late seventies, there were basically two types of algorithms for linear network optimization: the *simplex* method and its variations, and the *primal-dual* method and its close relative, the *out-of-kilter* method. There was some controversy regarding the relative merit of these methods, but thanks to the development of efficient implementation ideas, the simplex method emerged as the fastest of the two for most types of network problems.

A number of algorithmic developments in the eighties have changed significantly the situation. New methods were invented that challenged the old ones, both in terms of practical efficiency, and theoretical worst-case performance. Two of these methods, originally proposed by the author, called *relaxation* and *auction*, will receive a lot of attention in this book. The relaxation method, is a dual ascent method resembling the coordinate ascent method of unconstrained nonlinear optimization, that significantly outperforms in practice both the simplex and the primal-dual methods for many types of problems. Auction is a form of dual coordinate ascent method, based on the notion of ϵ-complementary slackness and scaling ideas. This algorithm together with its extensions, has excellent computational complexity, which is superior to that of the classical methods for many types of problems. Some auction algorithms have also proved to be very effective in practice, particularly for assignment and max-flow problems.

One of the purposes of the book is to provide a modern, and up-to-date synthesis of old and new algorithms for linear network flow problems. The coverage is focused and selective, concentrating on the algorithms that have proved most successful in practice or otherwise embody important methodological ideas. Two fundamental ideas of mathematical programming are emphasized: *duality* and *iterative cost improvement*. Algorithms are grouped in three categories: (a) *primal cost improvement methods, including simplex methods*, which iteratively improve the primal cost by moving flow around simple cycles, (b) *dual ascent methods*, which iteratively improve the dual cost by changing the prices of a subset of nodes by equal amounts, and (c) *auction algorithms*, which try to improve the dual cost approximately along coordinate directions.

The first two classes of methods are dual to each other when viewed in the context of Rockafellar's monotropic programming theory [Roc84]; they are based on cost improvement along elementary directions of the circulation space (in the primal case) or the differential space (in the dual case). Auction algorithms are fundamentally different; they have their origin in nondifferentiable optimization and the ϵ-subgradient method in particular [BeM73].

A separate chapter is devoted to each of the above types of methods. The introductory chapter establishes some basic material and treats a few simple problems such as max-flow and shortest path. A final chapter discusses some of the practical performance aspects of the various methods.

A second purpose of the book is to supply state-of-the-art FORTRAN codes based on some of the algorithms presented. These codes illustrate implementation techniques commonly used in network optimization and should be helpful to practitioners. The listings of the codes appear in appendixes at the end of the book, and are also available on diskette from the author. I am thankful to Giorgio Gallo and Stefano Pallotino who gave me permission to include two of their shortest path codes.

The book can be used for a course on network optimization or for part of a course on introductory optimization; such courses have flourished in engineering, operations research, and applied mathematics curricula. The book contains a large number of examples and exercises, which should enhance its suitability for classroom instruction.

I was fortunate to have several outstanding collaborators in my linear network optimization research and I would like to mention those with whom I have worked extensively. Eli Gafni programmed for the first time the auction algorithm and the relaxation method for assignment problems in 1979, and assisted with the computational exprerimentation. The idea of ϵ-scaling arose during my interactions with Eli at that time. Paul Tseng worked with me on network optimization starting in 1982. Together we developed the RELAX codes, we developed several extensions to the basic relaxation method, and we collaborated closely and extensively on a broad variety of other subjects. Paul also read a substantial part of the book and offered several helpful suggestions. Jon Eckstein worked with me on auction and other types of network optimization algorithms starting in 1986. Jon made several contributions to the theory of the ϵ-relaxation method, and coded its first implementation. Jon also proofread parts of the book and his comments resulted in several substantive improvements. David Castañon has been working with me on auction algorithms for assignment, transportation, and minimum cost flow problems since 1987. Much of our joint work on these subjects appears in Chapter 4, particularly in Sections 4.2 and 4.4. David and I have also collaborated extensively on the implementation of various network flow algorithms. Our interactions have resulted in several improvements in the codes of the appendixes.

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