# High-performance switchless WDM network using multiple free spectral ranges of an arrayed-waveguide grating 

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#### Abstract

In this paper, we report on a scalable and reliable switchless wavelength division multiplexing (WDM) network that is based on an arrayed-waveguide grating (AWG). All wavelengths are used for data transmission and signaling is done in-band. Each node at the network periphery is equipped with a single tunable transceiver for data and a broadband light source for control while the network itself is completely passive. Broadcasting is realized by spectrally slicing the broadband signal. The proposed random distributed medium access protocol is reservation based and schedules variably sized data packets on a first-come-first-served and first-fit basis without resulting collisions. The protocol supports both packet and circuit switching and allows for multicasting. The degree of concurrency is significantly increased by using multiple free spectral ranges (FSRs) of the AWG, spatially reusing wavelengths and transmitting data and control informations simultaneously by means of code division multiplexing. Our analytical results demonstrate that exploiting multiple FSRs of an AWG significantly improves the throughputdelay performance of the network.


Keywords: AWG, CDMA, MAC, multicasting, multiple FSRs, packet/circuit switching, reliability, scalability, spectral slicing, WDM

## 1. INTRODUCTION

Due to the very high transmission rates in optical wavelength division multiplexing (WDM) networks electronic processing devices are very expensive or even unfeasible at present resulting in electro-optic bottlenecks. Several photonic switching techniques have been investigated such as optical label switching (OLS), optical burst switching (OBS), optical packet switching (OPS) and photonic slot routing (PSR) ${ }^{1} .{ }^{2}$ Those approaches avoid electro-optic bottlenecks by letting the packets remain in the optical domain. WDM networks based on wavelength sensitive devices such as an AWG have routing characteristics that depend on the wavelength. Transmitters can reach different destinations by simply changing the wavelength. ${ }^{3}$ These networks naturally move the switching functionality towards the network periphery leading to significantly reduced network costs and complexity and simplified network management. With tunable transceivers we are able to realize switchless single-hop networks. ${ }^{4}$ Single-hop networks have some very desirable properties such as minimum hop distance (unity), high channel utilization, inherent transparency and low processing requirements at each node.

In this paper, we develop a novel MAC protocol for a switchless AWG based network. Our network architecture exploits multiple free spectral ranges (FSRs) of the AWG; thus increasing the network efficiency. We then develop and analyze a random Medium Access Control (MAC) protocol for the proposed network. Our MAC protocol uses spectrally sliced LEDs for in-band signaling and completely avoids collisions of data packets. We note that networks exploiting multiple FSRs of an AWG have received only little attention in the literature. To our knowledge this is the first paper to develop and analyze a random MAC protocol for a network exploiting multiple FSRs of an AWG.

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Figure 1. Properties of an AWG: a) Periodic wavelength routing, b) spatial wavelength reuse

## 2. UNDERLYING PRINCIPLES

In this section, we discuss two principles that are important for understanding the subsequent MAC protocol: (1) the periodic wavelength routing characteristics of an AWG and (2) the spectral slicing of a light emitting diode (LED) broadband signal. Without loss of generality we consider a $2 \times 2$ AWG.

### 2.1. Periodic Wavelength Routing

In Fig. 1 a) we show a scenario where six wavelengths are launched into the upper AWG input port. The AWG routes every second wavelength to the same output port. This period of the wavelength response is called free spectral range (FSR). In our example, there are three FSRs, each containing two wavelengths. Generally, the FSR of a $D \times D$ AWG, $D \in \mathbb{N}$, consists of $D$ wavelengths, i.e., the physical degree of an AWG is identical to the number of wavelengths per FSR. As shown in Fig. 1 b), this holds also for the lower AWG input port.

There are two important points to keep in mind. First, as shown in Fig. 1 b), each wavelength can be applied on all AWG input ports simultaneously. The AWG routes wavelengths such that no collisions occur at the AWG output ports. Thus, with a $D \times D$ AWG each wavelength can be spatially reused $D$ times. Second, each FSR provides one wavelength for communication between a given AWG input port and an arbitrary AWG output port. Hence, using $R$ FSRs, $R \in \mathbb{N}$, allows for $R$ simultaneous transmissions between each AWG input/output port pair.

### 2.2. Spectral Slicing

Spectral slicing is a means to realize broadcasting in WDM networks that are based on wavelength sensitive devices. ${ }^{5}$ Fig. 2 depicts the same scenario as shown above; in addition, an LED broadband signal is fed into the upper AWG input port. The LED signal spans all six wavelengths (channels). The AWG slices the LED spectrum such that in each FSR one slice is routed to either AWG output port. Using $R$ FSRs, there are $R$ slices at each AWG output port. All those slices carry the same information. Hence, receivers attached to the AWG output ports are free to choose one of the $R$ slices in order to retrieve the information. Note that while being tuned to any of those slices the receiver can monitor only wavelengths that originate from the same AWG input port as the LED signal. However,


Figure 2. Spectral slicing of a broadband LED signal


Figure 3. Network and node architecture
wavelengths and the LED broadband signal can be used simultaneously at the same AWG input port only if they can be distinguished at the receiver. This problem will be addressed in the next section.

There are three important points to keep in mind. First, spectral slicing is an elegant way to realize broadcasting. Secondly, each FSR contains one slice per AWG output port. Finally, listening to an LED slice restricts the receiver to wavelengths that originate from the same AWG input port as the LED signal.

## 3. ARCHITECTURE

The network and node architecture is depicted in Fig. 3. The network is based on a $D \times D$ AWG. At each AWG input port a wavelength-insensitive $S \times 1$ combiner is attached. Similarly, at each AWG output port signals are distributed by a wavelength-insensitive $1 \times S$ splitter. Each node is composed of a transmitting and receiving part. The transmitting part of a node is attached to one of the combiner ports. The receiving part of the same node is located at the opposite splitter port.

The network connects $N$ nodes, with $N=D \cdot S$. For a given number of nodes $N$ there are several possible architectures with different values of $D$ and $S$. The choice of $D$ and $S$ tradeoffs spatial wavelength reuse and receiver throughput. Due to the wavelength routing characteristics of the AWG each wavelength can be used at all ports simultaneously (spatial wavelength reuse). Spatial wavelength reuse increases the degree of concurrency resulting in an improved throughput-delay performance. Therefore, from the spectrum reuse point of view it is reasonable to choose a large $D$ for a given $N$. On the other hand, small values of $D$ imply that many receivers are attached to the same splitter, i.e., $S$ becomes large. This has the advantage that each packet can be received by more nodes leading to an increased receiver throughput. An increased receiver throughput allows for efficient multicasting since multicast packets have to be transmitted fewer times.

Let us now take a look at the node structure. Each node contains a laser diode (LD) for transmission and a photodiode (PD) for reception. Given the wavelength routing characteristics of the AWG, both transmitter and receiver have to be tunable over at least $D$ wavelengths in order to provide full connectivity. In addition, each node uses a light emitting diode (LED) for broadcasting control packets. The broadband LED signal ( $10-100 \mathrm{~nm}$ ) is spectrally sliced such that all receivers are able to obtain the control information. No additional receiver is required if the signaling is done in-band, i.e., LED and LD signals overlap spectrally. However, data and control information have to be distinguishable at the receiver. This can be achieved by code division multiple access (CDMA). ${ }^{6}$ The control information is spreaded before modulating the LED. Accordingly, at the receiving part the control information is retrieved by despreading a part of the incoming signal.

Although CDMA is a means to increase the degree of concurrency we use only one code. This reduces the crosstalk penalty and the computational complexity at each node. In the proposed architecture each node has to process the
control signals of all other $N-1$ nodes. This computational overhead can become a serious bottleneck that affects the network scalability. In order to accomodate a large number of nodes and make the entire network scalable it is important to keep the computational complexity at each node small.

Intrachannel crosstalk due to spatial wavelength reuse has also to be taken into account. ${ }^{7}$ As a consequence, the AWG has to be realized as a free-space device or integrated AWGs with only a relatively small physical degree can be deployed. However, in the next section we see that for a fixed channel spacing and transceiver tuning range AWGs with a small physical degree allow for the use of multiple FSRs resulting in an increased number of channels between each AWG input/output pair.

Another crucial issue is the small bandwidth-distance product of LEDs even though they are used for transmitting only low-rate control information. Especially for a large population the splitting losses due to the combiners and splitters in the proposed architecture put severe constraints on the power budget. Those constraints can be relaxed by inserting erbium-doped fiber amplifiers (EDFAs) between each combiner/splitter and the corresponding AWG port. Since the physical degree of the AWG is rather small only a few EDFAs would be required. Alternatively, each LED signal could be preamplified ${ }^{8}$ or other broadband light sources such as fiber amplifiers ${ }^{9}$ or Fabry-Perot lasers driven into clipping ${ }^{10}$ could be used instead of LEDs. Those solutions are either not very economic or support only small transmission rates. The most promising approach appears to be the use of superluminescent diodes (SLDs) which provide a significantly improved power budget. ${ }^{11}$

## 4. MAC PROTOCOL

The wavelength assignment is schematically shown in Fig. 4. The y-axis denotes the wavelengths used for transmission and reception. As illustrated, $R$ adjacent FSRs are exploited. Each FSR consists of $D$ contiguous channels, where $D$ denotes the physical degree of the underlying AWG. Transceivers are tunable over the range of $R \cdot D$ contiguous wavelengths. To avoid interferences at the receiver during simultaneous transmissions in different FSRs of the AWG, the FSR of the receivers have to be different. In our case, the FSR of a receiver is equal to $R \cdot D$ wavelengths. The x-axis denotes the time. Time is divided into cycles which are repeated periodically. Nodes are assumed to be synchronized to the cycle boundaries. Each cycle is further subdivided into $D$ frames.

The frame format of one wavelength is depicted in Fig. 5. A frame contains $F \in \mathbb{N}$ slots with a slot length equal to the transmission time of a control packet (function and format of a control packet will be explained later). The transceiver tuning time is assumed to be negligible. This is due to the fact that in the considered architecture the physical degree of the AWG is chosen above a certain threshold. This guarantees spatial wavelength reuse that is high enough to significantly reduce the wavelength pool and thereby the required transceiver tuning range. Transceivers with a limited tuning range such as electro-optic transceivers exhibit a negligible tuning time of a few nanoseconds. Each frame is partitioned into the first $M, 1 \leq M<F$, slots (shaded region) and the remaining ( $F-M$ ) slots. In the first $M$ slots the pretransmission coordination takes place. During this period control packets are transmitted and all nodes are obliged to tune their receivers to one of the corresponding LED slices (channels) in order to obtain the control information. Owing to the wavelength routing properties of the AWG, in a given frame only nodes that are attached to the same corresponding combiner can transmit control packets. Nodes attached to AWG input port $i$ (via a common combiner) send their control packets in frame $i, 1 \leq i \leq D$ (see Fig. 4). Each frame within a cycle accomodates control packets originating from a different AWG input port. Hence, after $D$ frames (one cycle) all nodes have had the opportunity to send their control packets guaranteeing fairness. The $M$ slots are not fixed assigned. Instead, control packets are sent on a contention basis using slotted ALOHA. This makes the entire network scalable. Control packets arrive at the receivers after a propagation delay that is equal to half the round-trip time. In the last $(F-M)$ slots of each frame no control packets are sent, allowing receivers to be tuned to any arbitrary wavelength. This freedom enables transmissions between any pair of nodes. During those slots each node processes the received control packets by executing the same scheduling algorithm. The parameter $M$ tradeoffs two kinds of concurrency. During the first $M$ slots of each frame, control and data packets can be transmitted simultaneously, but only from nodes which are attached to the same AWG input port. In this time interval packets originating from other AWG input ports cannot be received. Whereas, during the last $(F-M)$ slots of each frame all receivers are unlocked and can be tuned to any arbitrary wavelength. As a consequence, during this time interval data packets from any AWG input port can be received. This allows for spatial wavelength reuse.

The MAC protocol works as follows. First, we consider the transmitters at each node. If a node has no data packet in its buffer the LED and LD remain idle. When a data packet destined to node $j, 1 \leq j \leq N$, arrives at


Figure 4. Wavelength assignment


Figure 5. Frame format
node $i \neq j, 1 \leq i \leq N$, node $i$ 's LED broadcasts a control packet in one of the $M$ slots of the frame allocated to the AWG input port that node $i$ is attached to. The slot is chosen randomly according to a uniform distribution. A control packet consists of three fields, namely, destination address, length of the corresponding data packet, and a type field. As illustrated in Fig. 5, the data packet can be of variable size $L, 1 \leq L \leq F$, where $L$ denotes the length in units of slots. The type field contains one bit and is used to enable packet and circuit switching.

Let us now take a look at the tunable receiver at each node. Every node collects all control packets by tuning its receiver to one of the corresponding channels during the first $M$ slots of each frame. Thus, it learns about all other nodes' activities and whether its own control packet was successful or not. In frame $k, 1 \leq k \leq D$, each receiver collects the control packets originating from nodes that are attached to AWG input port $k$. If its control packet has collided, node $i$ retransmits the control packet in the next cycle with probability $p$, and with probability $(1-p)$ it will defer the transmission by one cycle. The node retransmits the control packet in this next cycle with probability
$p$, and so forth. Successful control packets are put in a distributed queue.
All nodes process the successful control packets by executing the same arbitration (scheduling) algorithm in the last $(F-M)$ slots of each frame. Consequently, all nodes come to the same transmission and reception schedule. Note that $M$ has to be smaller than $F$ to provide enough time for executing the algorithm. Since each node has to process the control packets of all nodes the computational complexity at each node puts severe constraints on the network scalability. A simple arbitration algorithm is required to relax those constraints. For now, we apply a straightforward greedy algorithm which schedules the control packets on a first-come-first-served and first-fit basis. After receiving a successful control packet the arbitration algorithm tries to schedule the transmission of the corresponding data packet within the following $D$ frames. Those $D$ frames do not necessarily have to coincide with the cycle boundaries. The data packet is sent in the first possible slot(s) using the lowest available wavelength. If there are not enough slots available within the $D$ frames the data packet is not transmitted and the source node has to retransmit the control packet in the next cycle.

The length of the scheduling window is equal to $D$ for two reasons. First, by limiting the scheduling length to a small number of frames (recall that $D$ is expected to be rather small) the computational requirements at each node are kept small as well. Secondly, every $D$ frames all nodes receive control packets from the same set of nodes. At the same time, due to the wavelength routing properties of the AWG and the requirement that all nodes listen to the LED slices only this set of nodes can transmit data packets. Those data packets were announced by control packets exactly $D$ frames earlier. By making the scheduling window $D$ frames long, data and control packets can be sent simultaneously. This kind of concurrency leads to an improved throughput-delay performance.

Next, we discuss the support for multicasting and circuit switching. Multicasting is realized by the splitters. Each splitter distributes an incoming packet to all attached nodes. By tuning the receivers to the respective wavelength the packet can be obtained by more than one node. The resulting increased receiver throughput has a positive impact on the network performance. Circuit switching is realized by using the type and length fields of the control packet. The length field denotes the required number of slots per cycle. By setting the bit in the type field the source node indicates that this number of slots must be reserved in each cycle. After receiving the control packet the circuit is set up by choosing the first possible free slots at the lowest available wavelength. Those slots are reserved in the subsequent cycles until the connection is terminated. If there are not enough free resources the control packet is discarded and has to be retransmitted in the next cycle. The termination of a circuit works as follows. Suppose node $i, 1 \leq i \leq N$, has set up a circuit, i.e., node $i$ is granted a certain number of slots per cycle which was specified in the foregoing control packet. Furthermore, suppose $j, 1 \leq j \leq M-1$, other nodes attached to the same combiner hold currently circuits. Then, in each cycle node $i$ repeats the control packet in slot $j+1$ of the corresponding reservation window. To terminate the circuit, node $i$ simply stops repeating the control packet. In doing so, all other nodes notice that the circuit has terminated and the respective slot is freed up for contention. Note that during the holding time of a circuit other circuits can be torn down. As a consequence, the corresponding slot, say $k, 1 \leq k \leq j+1$, becomes idle. Whenever this happens, all slots which are larger than $k$ and are used to indicate the existence of circuits are decremented by one. Thus, the first $j$ slots of the corresponding reservation window indicate the existence of circuits while the remaining $(M-j)$ slots are free to be used for reservations. A node with a control packet to send chooses randomly one of slots $j+1, j+2, \ldots, M$.

## 5. MODEL

In our analysis we focus on the case of fixed size data packets. For the analysis we make the following typical assumptions ${ }^{12}$ :

- A node with an empty buffer generates a data packet with probability $\sigma$ at the end of a frame.
- Each node has a single-packet buffer. After transmitting a data packet in a given frame the buffer becomes empty at the end of that frame.
- A data packet has a fixed size of $F$ slots, i.e., $L=F$.
- Uniform unicast traffic: A data packet is destined to any one of the other $N-1$ nodes with equal probability.
- A transmitting node holds a circuit with probability $r$. Thus, $r$ denotes the fraction of traffic that is circuit switched.


Figure 6. Model for MAC protocol

- The propagation delay $\tau$ is the same for all nodes and is an integer multiple of one frame, i.e., all nodes are equidistant from the AWG.
- Nonpersistency ${ }^{13}$ : Random selection of a destination node among the other $N-1$ nodes is renewed for each attempt of transmitting a control packet. (The nonpersistency assumption is needed to obtain a Markovian model.)
- Delayed first-time transmission ${ }^{12}$ : A node sends out its control packet in a frame with probability $p$, for both first-time transmission as well as retransmissions. (This assumption simplifies the calculation of the probability of control packet collisions.)

Fig. 6 depicts an approximate model for the MAC protocol. Each node can be in one of the $(2 \tau+3)$ modes during any frame. Transitions from one mode to another mode occur only at the beginning of a frame. The modes are defined as follows:

- TH: Nodes in the TH (thinking) mode generate a data packet with probability $\sigma$ at the end of a frame.
- B: Nodes in this mode are backlogged and send a control packet with probability $p \cdot \beta$ (where $\beta$ accounts for the cycles in the time structure, as shall be explained later) at the beginning of the next frame.
- $P Q_{1}, P Q_{2}, \ldots, P Q_{\tau}$ : Those modes represent the propagation delay. After successfully transmitting a control packet the corresponding data packet is put in the distributed queue. Nodes move from mode $P Q_{i}$ to mode $P Q_{i+1}, i=1,2, \ldots, \tau-1$, at the beginning of the next frame with probability 1 .
- $P R_{1}, P R_{2}, \ldots, P R_{\tau}$ : Those modes are similar to the $P Q_{i}$ modes, $i=1,2, \ldots, \tau$. Nodes whose collided control packets have to be retransmitted enter the mode $B$ after $\tau$ frames.
- $T R$ : Nodes in the mode $P Q_{\tau}$ whose data packets are successfully scheduled move to mode $T R$ (transmission). After one frame those nodes return to the $T H$ mode. Nodes in mode $P Q_{\tau}$ whose data packets are not scheduled due to the lack of free resources (not enough free slots and/or wavelengths) move to mode $B$.

The system state in frame $n, n \in \mathbb{Z}$, is completely described by the following state vector:

$$
\mathbf{N}(n)=\left(N_{B}(n), N_{P R_{1}}(n), \ldots, N_{P R_{\tau}}(n), N_{P Q_{1}}(n), \ldots, N_{P Q_{\tau}}(n), N_{T R}(n)\right)
$$

where $N_{X}(n)$ denotes the number of nodes in mode $X$ in frame $n$. Note that $N_{T H}$ is not included in the state vector since it is linearly dependent on the other modes. With the nonpersistency assumption

$$
\{\mathbf{N}(0), \mathbf{N}(1), \ldots, \mathbf{N}(n), \ldots\}
$$

is a discrete-time multi-dimensional Markov chain with finite but quite large state space. The exact analysis of that Markov chain would involve the calculation of the state transition probability matrix which is computationally prohibitive. Therefore, we analyze the system at an equilibrium point using the equilibrium point analysis (EPA) approach. ${ }^{12}$

## 6. ANALYSIS

In the EPA method the system is assumed to be always at an equilibrium point, defined as

$$
\mathbf{N}=\left(N_{B}, N_{P R_{1}}, \ldots, N_{P R_{\tau}}, N_{P Q_{1}}, \ldots, N_{P Q_{\tau}}, N_{T R}\right) .
$$

At an equilibrium point the expected increase in the number of nodes in each mode per unit time (i.e., frame) is zero. Applying this condition to all the modes, we get a set of so-called equilibrium point equations.

### 6.1. Equilibrium Point Equations

By writing the equation for each mode, we get $(2 \tau+3)$ equations. Let $\delta_{X}(\mathbf{N})$ be the conditional expectation of the increase in the number of nodes in mode $X$ in a frame, given that the system is in state $\mathbf{N}$. Since $\delta_{P R_{i}}(\mathbf{N})=$ $N_{P R_{i-1}}-N_{P R_{i}}=0, i=2,3, \ldots, \tau$, we can omit the subscript of $P R$ by letting

$$
\begin{equation*}
N_{P R}=N_{P R_{1}}=\cdots=N_{P R_{\tau}} \tag{1}
\end{equation*}
$$

Similarly, for the modes $P Q_{i}, i=1,2, \ldots, \tau$, we get

$$
\begin{equation*}
N_{P Q}=N_{P Q_{1}}=\cdots=N_{P Q_{\tau}} . \tag{2}
\end{equation*}
$$

For the modes $T H$ and $B$ we have the following equations:

$$
\begin{align*}
\delta_{T H}(\mathbf{N}) & =N_{T R}-N_{T H} \sigma \\
& =N_{T R}-\left[N-N_{B}-\tau\left(N_{P R}+N_{P Q}\right)-N_{T R}\right] \sigma=0  \tag{3}\\
\delta_{B}(\mathbf{N}) & =\left[N_{T H} \sigma+\left(N_{P Q}-N_{T R}\right)+N_{P R}\right]-N_{B} p \beta=0 . \tag{4}
\end{align*}
$$

To obtain the remaining equilibrium point equations for the modes $P R_{1}, P Q_{1}$ and $T R$ we introduce the quantities $Y(\mathbf{N})$ and $Z(\mathbf{N})$. Let $Y(\mathbf{N})$ denote the conditional expectation of the number of nodes that move out from mode $B$ to mode $P Q_{1}$, given that the system is in state $\mathbf{N} . Y(\mathbf{N})$ is identical to the average number of control packets transmitted in a frame without collision. With $Y(\mathbf{N})$ we obtain the following equations for the modes $P R_{1}$ and $P Q_{1}$ :

$$
\begin{align*}
\delta_{P R_{1}}(\mathbf{N}) & =N_{B} p \beta-Y(\mathbf{N})-N_{P R}=0  \tag{5}\\
\delta_{P Q_{1}}(\mathbf{N}) & =Y(\mathbf{N})-N_{P Q}=0 \tag{6}
\end{align*}
$$

Let $Z(\mathbf{N})$ denote the conditional expectation of the number of nodes that move from mode $P Q_{\tau}$ to mode $T R$, given that the system is in state $\mathbf{N} . Z(\mathbf{N})$ is identical to the average number of nodes that successfully transmit a data packet in a frame. With $Z(\mathbf{N})$ we obtain the following equation for the mode $T R$ :

$$
\begin{equation*}
\delta_{T R}(\mathbf{N})=Z(\mathbf{N})-N_{T R}=0 . \tag{7}
\end{equation*}
$$

Next, we need to solve for the unknown quantities $\beta, Y(\mathbf{N})$ and $Z(\mathbf{N})$. Recall that a backlogged node can transmit a control packet only in one frame per cycle that consists of $D$ frames. Let $\beta$ denote the probability that the next frame is allocated to the backlogged node. Thus, we get

$$
\begin{equation*}
\beta=\frac{1}{D} \tag{8}
\end{equation*}
$$

On the average, $N_{T R} \cdot r$ nodes transmit a control packet in each frame to indicate the existence of the corresponding circuits. Consequently, there are $\left(M-N_{T R} \cdot r\right)$ free reservation slots in each frame and the average number of successfully transmitted control packets per frame is given by ${ }^{12}$

$$
\begin{align*}
Y(\mathbf{N}) & =\sum_{i=1}^{N_{B}} i\left(1-\frac{1}{M-N_{T R} \cdot r}\right)^{i-1}\binom{N_{B}}{i}(p \beta)^{i}(1-p \beta)^{N_{B}-i}  \tag{9}\\
& =N_{B} p \beta\left(1-\frac{p \beta}{M-N_{T R} \cdot r}\right)^{N_{B}-1} \tag{10}
\end{align*}
$$

The result can be interpreted such that $p \beta\left(1-\frac{p \beta}{M-N_{T R} \cdot r}\right)^{N_{B}-1}$ is the probability that a node's control packet is transmitted collisionfree. The average number of nodes that move from mode $B$ to mode $P Q_{1}$ is given by equation (10).

Let $q$ be the probability that a given slot of the first $M$ slots of a frame contains exactly one control packet that is to be scheduled. Then,

$$
\begin{equation*}
q=\frac{Y(\mathbf{N})+N_{T R} \cdot r}{M} . \tag{11}
\end{equation*}
$$

The probability that exactly $i$ control packets are to be scheduled in a frame is

$$
\begin{equation*}
P_{i}=\binom{M}{i} q^{i}(1-q)^{M-i} \quad, i=0,1,2, \ldots, M \tag{12}
\end{equation*}
$$

Each of the $i$ control packets originates from one of the $N$ nodes with equal probability $1 / N$. With $i$ control packets, in each frame the average number of control packets that belong to nodes attached to the same combiner is equal to $i \cdot \beta=i / D$. Recall that due to their fixed size of $F$ slots, data packets can be sent from those nodes only every $D$ frames. Data packets cannot be transmitted in other frames since they are larger than $F-M$ slots. Thus, in each frame only nodes attached to the same combiner can send data packets. Control packets emanating from nodes attached to the same combiner aggregate over the interval of $D$ frames until data transmission takes place. As a consequence, in each frame the average number of control packets to be scheduled is given by $i / D \cdot D=i$.

The probability that at least one among those $i$ control packets is destined to a given node under the assumption that a node does not transmit to itself is equal to ${ }^{13}$

$$
\begin{align*}
p_{o}(i) & =1-\left[\frac{i}{N}\left(1-\frac{1}{N-1}\right)^{i-1}+\left(1-\frac{i}{N}\right)\left(1-\frac{1}{N-1}\right)^{i}\right]  \tag{13}\\
& =1-\left(1-\frac{1}{N-1}\right)^{i-1} \frac{N^{2}-2 N+i}{N(N-1)} \tag{14}
\end{align*}
$$

Let $g(i)$ denote the average number of nodes that successfully transmit a data packet in a frame, given that $i$ control packets are to be scheduled. Given this, the number of data packets destined to nodes that are attached to the same splitter is binomially distributed $B\left(S, p_{o}(i)\right)$, with an average value equal to $\sum_{k=0}^{S} k\binom{S}{k} p_{o}(i)^{k}\left[1-p_{o}(i)\right]^{S-k}$. However, no more than $R$ data packets can be simultaneously transmitted to those nodes. This holds for each of the $D$ splitters and we finally obtain

$$
\begin{equation*}
g(i)=D\left\{\sum_{k=0}^{R} k\binom{S}{k} p_{o}(i)^{k}\left[1-p_{o}(i)\right]^{S-k}+R \sum_{k=R+1}^{S}\binom{S}{k} p_{o}(i)^{k}\left[1-p_{o}(i)\right]^{S-k}\right\} . \tag{15}
\end{equation*}
$$

Note that at most $S$ nodes can transmit data packets in a frame. Hence, $g(i)$ is bounded and the number of actually transmitting nodes is equal to $\min \{g(i), S\}$.

The conditional expectation of the number of nodes that successfully transmit a data packet in a frame, given that the system is in state $\mathbf{N}$, is given by

$$
\begin{align*}
Z(\mathbf{N})= & \sum_{i=0}^{M} g(i) \cdot P_{i}  \tag{16}\\
= & \sum_{i=0}^{M} D\left\{\sum_{k=0}^{R} k\binom{S}{k} p_{o}(i)^{k}\left[1-p_{o}(i)\right]^{S-k}+R \sum_{k=R+1}^{S}\binom{S}{k} p_{o}(i)^{k}\left[1-p_{o}(i)\right]^{S-k}\right\} \\
& \cdot\binom{M}{i} q^{i}(1-q)^{M-i} . \tag{17}
\end{align*}
$$

Using equations (5)-(8) we can modify equations (3) and (10). Equation (3) becomes

$$
\begin{equation*}
Z(\mathbf{N})=\frac{\sigma}{1+\sigma}\left[N-\left(1+\frac{\tau p}{D}\right) N_{B}\right] \tag{18}
\end{equation*}
$$

and equation (10) becomes

$$
\begin{equation*}
Y(\mathbf{N})=N_{B} \frac{p}{D}\left(1-\frac{p}{D(M-Z(\mathbf{N}) \cdot r)}\right)^{N_{B}-1} \tag{19}
\end{equation*}
$$

Equations (17), (18) and (19) can be solved simultaneously for the variables $N_{B}, Y(\mathbf{N})$ and $Z(\mathbf{N})$. The system is unstable if there is more than one solution. Otherwise, if only one solution exists, the system is stable. $N_{B}, Y(\mathbf{N})$ and $Z(\mathbf{N})$ can then be used to provide the steady-state solution of the entire system. $N_{P R_{i}}, i=1,2, \ldots, \tau$, is given by equations (1) and (5). Similarly, $N_{P Q_{i}}, i=1,2, \ldots, \tau$, is given by equations (2) and (6). According to equation (7), $N_{T R}$ is equal to $Z(\mathbf{N})$. And $N_{T H}$ equals $N$ minus the sum of the nodes in all other modes.

### 6.2. Performance Measures

The performance measures of interest are throughput and delay at an equilibrium point. The throughput $S(\mathbf{N})$ is defined as the expected number of nodes in the active mode $T R$ :

$$
\begin{equation*}
S(\mathbf{N})=N_{T R} \tag{20}
\end{equation*}
$$

The mean packet delay $D(\mathbf{N})$ is measured from the time the packet is generated at a node until the end of the frame during which it is transmitted. The system shown in Fig. 6 is a closed system, and, by Little's law, $N / S(\mathbf{N})$ is the average time that a packet experiences from the moment the packet enters mode $T H$ until the time it returns to mode $T H$. Also, $1 / \sigma$ is the average time that a packet stays in mode $T H$. Thus, we get the average packet delay as

$$
\begin{equation*}
D(\mathbf{N})=\frac{N}{S(\mathbf{N})}-\frac{1}{\sigma} \tag{21}
\end{equation*}
$$

Note that $D(\mathbf{N})$ is measured in number of frames.

## 7. NUMERICAL RESULTS

Due to space limitations we investigate the impact of the key parameters on the throughput-delay performance of the network. In particular we consider propagation delay $\tau$, physical degree of the AWG $D$, number of FSRs $R$, and number of reservation slots per frame $M$. Unless stated otherwise, the parameters are set to the following default values: $D=2, M=8, N=240, p=0.5, r=0.2, R=3, S=120, \tau=10$.

Fig. 7 depicts the impact of the propagation delay on the network performance. At low traffic loads packets experience less delay for smaller propagation delays. Whereas with increasing traffic larger propagation delays provide a better throughput-delay performance. This is due to the fact that at low traffic loads almost no collisions of control packets occur and nodes receive the successfully transmitted control packets earlier with smaller propagation delays resulting in a decreased delay. At higher traffic loads the control channel gets more congested. In this case, a larger propagation delay implies that nodes have to wait a longer time interval for the transmitted control packets. During this time period those nodes do not access the control channel resulting in less contention and an increased throughput and a decreased delay due to fewer retransmissions.

Recall that a given number of nodes can be connected by AWGs with different physical degrees. Fig. 8 shows that a better throughput-delay performance can be achieved by choosing AWGs with high physical degrees. In this case, fewer nodes are attached to each combiner resulting in a reduced number of collisions on the control channel. However, a higher physical degree implies a longer cycle, i.e., more nodes are likely to be backlogged and access the control channel leading to more collisions. This is why the performance improvement weakens with increasing physical degree as illustrated in the figure. At light traffic, AWGs with small degrees provide a slightly smaller delay since a backlogged node has to wait less time for the assigned frame due to the shorter cycle length.

The results in Fig. 9 clearly demonstrate the benefit of using multiple FSRs of an AWG. Each additional FSR increases the degree of concurrency and thereby significantly improves the throughput-delay performance of the network. Using three FSRs instead of one, which is typically done in the literature, roughly doubles the maximum throughput. However, using multiple FSRs requires transceivers with a larger tuning range.

Fig. 10 shows that a large reservation window has a positive impact on the system performance. By using many reservation slots the collision probability in each slot is reduced and the number of successful control packets is increased. However, long reservation windows require large data packets. This could be realized by means of traffic aggregation (grooming). Note that for $M=14$ all three FSRs are used for data transmission.


Figure 7. Mean packet delay vs. throughput for different propagation delays $\tau \in\{5,10,15\}$


Figure 9. Mean packet delay vs. throughput for different numbers of FSRs $R \in\{1,2,3\}$


Figure 8. Mean packet delay vs. throughput for different AWG degrees $D \in\{2,3,4\}$


Figure 10. Mean packet delay vs. throughput for different numbers of reservation slots $M \in\{6,10,14\}$

## 8. CONCLUSION AND FUTURE WORK

In this paper, we have proposed and investigated an AWG based switchless WDM network with a very high degree of concurrency. All wavelengths are used for data transmission and in-band signaling is deployed, i.e., no additional control channel is required. The node structure is simple and economical. Each node is equipped with a single tunable transceiver and an LED. All nodes have global knowledge at any time and by executing a common distributed deterministic scheduling algorithm network resources are used efficiently. The MAC protocol completely avoids collisions of data packets resulting in an improved throughput-delay performance. The network consists of passive components and is scalable. Our analytical results indicate that the use of multiple FSRs of an AWG considerably improves the throughput-delay performance of the network.

Future work will focus on the more general case of variable-size packets. Beside in-band signaling and using multiple FSRs spatial wavelength reuse will be taken into account as well. Spatial wavelength reuse is expected to further improve the throughput-delay performance of the network. In addition, multicasting will be incorporated in order to increase the efficiency and the receiver throughput. Finally, we plan to formulate an optimization problem with the objective of maximizing throughput and/or minimizing mean packet delay.

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