# Shortest Path Routing in Optical WDM Ring Networks under Multicast Traffic (Extended Version)

Michael Scheutzow, Patrick Seeling, Martin Maier, and Martin Reisslein

**Abstract**

We present an analytical model to investigate the mean hop distance of shortest path routing bidirectional optical WDM ring networks not only for multicast traffic with arbitrary fanout but also for unicast and broadcast traffic.

**Index Terms**

Mean hop distance, multicasting, optical ring network, shortest path routing, wavelength division multiplexing (WDM).

## I. Introduction

Optical bidirectional rings typically deploy shortest path routing in order to minimize the number of required hops and to maximize thereby the number of simultaneous transmissions and network capacity by means of spatial reuse of bandwidth resources, as done for instance in IEEE 802.17 Resilient Packet Ring (RPR). In this paper, we analytically investigate shortest path routing in terms of mean hop distance for optical bidirectional wavelength division multiplexing (WDM) rings under multicast as well as unicast and broadcast traffic.

## II. WDM Ring Network

We consider a bidirectional WDM ring network with the set of wavelength channels $\lambda \in \{1, 2, \ldots, \Lambda\}$ on the clockwise fiber ring and the same set $\lambda \in \{1, 2, \ldots, \Lambda\}$ on the counterclockwise fiber ring. The network interconnects $N$ nodes, which we index without loss of generality sequentially in the clockwise direction as $n = 1, 2, \ldots, N$. We consider node structures where each node $(i)$ can transmit on any wavelength, and $(ii)$ receive on one (home) wavelength using a single fixed-tuned receiver (FR), whereby

M. Scheutzow is with the Department of Mathematics, Technical University Berlin, 10623 Berlin, Germany (e-mail: ms@math.tu-berlin.de).

P. Seeling is with the Department of Electrical Engineering, Arizona State University, Tempe, AZ 85287–5706, USA (e-mail: patrick.seeling@asu.edu).

M. Maier is with the Institut National de la Recherche Scientifique (INRS), Montréal, QC, H5A 1K6, CANADA (e-mail: maier@ieee.org).

Corresponding Author: M. Reisslein is with the Department of Electrical Engineering, Arizona State University, Tempe, AZ 85287–5706, USA (e-mail: reisslein@asu.edu, web: http://www.fulton.asu.edu/˜mre, phone: (480) 965–8593, fax: (480) 965–8325).

the nodes $n = \lambda + k \cdot \Lambda$ with $k = 0, 1, \ldots, (\eta - 1)$ with $\eta := N/\Lambda$ (which we assume to be an integer) share the same home wavelength $\lambda$, $\lambda = 1, 2, \ldots, \Lambda$.

## III. Analysis

### A. Traffic Model

We consider the distribution of the number $F$ of destination nodes for a given packet to be given by

$$\mu_l = P(F = l), \quad l = 1, 2, \ldots, N - 1. \tag{1}$$

### B. Evaluation of Hop Distance

The total expected hop distance $E[H]$ required to serve a given multicast packet is obtained by summing the expected hop distances $E[H_\lambda]$ required on the individual wavelengths, i.e.,

$$E[H] = \sum_{\lambda=1}^{\Lambda} E[H_\lambda]. \tag{2}$$

We evaluate the expected hop distance $E[H_\lambda]$ by extending the notion of the largest gap, detailed in [1] for the single-wavelength ring network, to the WDM ring as follows. For each wavelength $\lambda$, $\lambda = 1, \ldots, \Lambda$, we define the largest gap as the largest hop distance between any two successive nodes (on the perimeter of the ring) in the set of nodes containing the source node (which we assume without loss of generality to be homed on wavelength $\Lambda$) and all $\ell$ multicast destination nodes (out of a total of $l$ multicast destinations) that are homed on the considered wavelength $\lambda$. Let $g^{(\lambda)}_{\ell,\eta}$ denote the conditional expectation of the length of the largest gap (in number of hops) between the considered set of nodes for a wavelength $\lambda$ given that there are $\ell$ multicast destinations on the wavelength. Note that with shortest path routing, the multicast packet(s) on a given wavelength traverse the entire ring, except for the largest gap, i.e., the hop distance on a wavelength $\lambda$ homing $\ell$ multicast destinations is given by $N - g^{(\lambda)}_{\ell,\eta}$. We also note that given a total of $l$ multicast destinations, the conditional probability for $\ell$ of these destinations being homed on wavelength $\lambda \neq \Lambda$ (with $\Lambda$ homing the source node), is given by

$$P(F_{\Lambda,\lambda} = \ell | F = l) = \frac{\binom{\eta}{\ell}\binom{N-1-\eta}{l-\ell}}{\binom{N-1}{l}} \tag{3}$$

and the conditional probability for $\ell$ destinations being homed on $\Lambda$ is given by

$$P(F_{\Lambda,\Lambda} = \ell | F = l) = \frac{\binom{\eta-1}{\ell}\binom{N-\eta}{l-\ell}}{\binom{N-1}{l}}, \tag{4}$$

see [2] for details. With these conditional probabilities we obtain

$$E[H_\lambda] = \begin{cases} \sum_{l=1}^{N-1} \mu_l \left[\sum_{\ell=1}^{\eta} P(F_{\Lambda,\lambda} = \ell | F = l) \cdot (N - g^{(\lambda)}_{\ell,\eta})\right] & \text{for } \lambda \neq \Lambda \\ \sum_{l=1}^{N-1} \mu_l \left[\sum_{\ell=1}^{\eta-1} P(F_{\Lambda,\Lambda} = \ell | F = l) \cdot (N - g^{(\Lambda)}_{\ell,\eta})\right] & \text{for } \lambda = \Lambda. \end{cases} \tag{5}$$

*1) Conditional Expectation $g^{(\Lambda)}_{\ell,\eta}$:* We can express the expected length $g^{(\Lambda)}_{\ell,\eta}$ of the largest gap on wavelength $\Lambda$ of the WDM ring in terms of the expected length of the largest gap $g(\ell,\eta)$ on a single-wavelength ring analyzed in [1] by observing that the only difference between the two is that successive nodes on wavelength $\Lambda$ in the WDM ring are spaced $\Lambda$ hops apart, whereas successive nodes in the single-channel ring are one hop apart. Hence, $g^{(\Lambda)}_{\ell,\eta} = \Lambda \cdot g(\ell,\eta)$, whereby $g(\ell,\eta)$ is given by Eqn. (16) in [1] as $g(\ell,\eta) = \sum_{k=1}^{\eta-1} k \cdot q_{\ell,\eta}(k)$ with $q_{\ell,\eta}(\cdot)$ denoting the distribution of the length of the largest gap and being calculated from the recursion $q_{\ell,\eta}(k) = p_{\ell,\eta}(k) \cdot \sum_{m=1}^{k} q_{\ell-1,\eta-k}(m) + \sum_{m=1}^{k-1} p_{\ell,\eta}(m) \cdot q_{\ell-1,\eta-m}(k)$ with the initialization $q_{0,\eta}(k) = 1$ for $k = \eta$ and $q_{0,\eta}(k) = 0$ for $k < \eta$ and with $p_{\ell,\eta}(k) = \binom{\eta-k-1}{\ell-1} / \binom{\eta-1}{\ell}$ denoting the probability that an arbitrary gap has $k$ hops.

*2) Evaluation of Conditional Expectation $g^{(\lambda)}_{\ell,\eta}$ for wavelengths $\lambda \neq \Lambda$:* The wavelengths $\lambda$, $\lambda \neq \Lambda$, in the WDM ring have the additional distinctions from the single-channel ring that $(i)$ the source node of the multicast is not homed on the wavelength $\lambda$, and $(ii)$ the hop distances from a source node on wavelength $\Lambda$ to the multicast destinations on wavelength $\lambda$, $\lambda \neq \Lambda$, are not integer multiples of $\Lambda$. Instead, the hop distances are generally $\lambda + j\Lambda$ in the clockwise direction and $(j+1)\Lambda - \lambda$ with $j = 0, \ldots, \eta - 1$ in the counter clockwise direction.

We employ the following analytical strategy for evaluating the conditional expectation $g^{(\lambda)}_{\ell,\eta}$ of the length of the largest gap on the wavelengths $\lambda$, $\lambda \neq \Lambda$. We initially leave the source node out of consideration and analyze the largest *spacing* (in number of hops) between successive multicast destinations on wavelength $\lambda$. We then introduce the source node and analyze how the placement of the source node affects the expected lengths of the two largest destination node spacings and the resultant largest gap. In brief, if the source node falls outside the largest spacing, then the largest spacing is equivalent to the largest gap. If the source node falls inside the largest spacing, it subdivides the spacing and depending on the size of the subdivisions, one of the subdivisions or the second largest spacing becomes the largest gap.

Formally, let $S_1, S_2, \ldots, S_\ell$ be random variables denoting the lengths of the spacings between successive multicast destinations on wavelengths $\lambda$, $\lambda \neq \Lambda$, in hops, and note that these random variables take on values that are integer multiples of $\Lambda$. Note that these spacings are identically distributed and let $S_{\max,1}$ be a random variable denoting the length of the largest spacing, i.e., $S_{\max,1} := \max\{S_1, \ldots, S_\ell\}$. Noting that the spacings of the $\ell$ multicast destinations among the $\eta = N/\Lambda$ nodes on wavelength $\lambda$ are equivalent to the gaps on a single wavelength ring homing $\eta$ nodes of which $\ell - 1$ nodes are multicast destinations and one node is the source node of the multicast, we have

$$P(S_{\max,1} = k \cdot \Lambda) = q_{\ell-1,\eta}(k), \tag{6}$$

whereby $q(\cdot)$ is given by Eqn. (16) in [1] as reviewed above. Let $S_{\max,2}$ be a random variable denoting the length of the second largest spacing, i.e., $S_{\max,2} := \max\{\{S_1, \ldots, S_\ell\} \setminus \{S_{\max,1}\}\}$. Let $r_{\ell-1,\eta}$ denote the joint distribution of the largest and second largest spacing, i.e., $r_{\ell-1,\eta}(k,m) := P(S_{\max,1} = k\Lambda, S_{\max,2} = m\Lambda)$, which we evaluate next and is subsequently used to evaluate the distribution of the length of the largest gap.

*3) Evaluation of Joint Distribution $r_{\ell-1,\eta}(k,m)$:* We evaluate the joint distribution of the largest and second largest spacing $r_{\ell-1,\eta}(k,m)$ for $1 \leq m \leq k \leq \eta$ and for $\ell \geq 2$ (the case $\ell = 1$ is treated separately at the end of Section III-B.4) by considering the number of possible ways of placing the multicast destination nodes on wavelength $\lambda$ so as to achieve a largest spacing with $k\Lambda$ hops and a second largest spacing with $m\Lambda$ hops. Note that there are a total of $\binom{\eta}{\ell}$ possibilities for selecting the $\ell$ multicast destinations out of the $\eta$ nodes homed on $\lambda$. For $m < k$ we can write

$$r_{\ell-1,\eta}(k,m) = \frac{\#\text{ of ways to select } \ell \text{ dest. out of } \eta \text{ nodes such that } S_{\max,1} = k\Lambda \text{ and } S_{\max,2} = m\Lambda}{\binom{\eta}{\ell}} \tag{7}$$

$$= \frac{\eta \cdot \#\text{ of ways to select } \ell-2 \text{ dest. out of } \eta-k-1 \text{ nodes s.t. largest spac. has } m\Lambda \text{ hops}}{\binom{\eta}{\ell}} \tag{8}$$

$$= \frac{\eta \cdot q_{\ell-2,\eta-k}(m) \cdot \binom{\eta-k-1}{\ell-2}}{\binom{\eta}{\ell}}, \tag{9}$$

where (8) follows by noting that there are $\eta$ possible positions for the largest gap. Equation (9) follows by noting the following two points: (A) The largest gap is bordered by two destination nodes, thus there are $\ell - 2$ nodes left to position after the position of the largest gap is fixed. There are $k + 1$ nodes that are inside or border on the largest gap of $k\Lambda$ hops, thus there are $\eta - (k + 1)$ nodes left to position the remaining $\ell - 2$ destinations on. Hence, there are $\binom{\eta-k-1}{\ell-2}$ ways for this positioning of the remaining $\ell - 2$ nodes. (B) Merging the endpoints of the largest gap with $k\Lambda$ hops into one destination node results in a ring with $\eta - k$ nodes on which $\ell - 1$ destination nodes are placed. The event that the largest spacing on this ring with a total of $\eta - k$ nodes and $\ell - 1$ destination nodes has $m\Lambda$ hops is equivalent to the event that the largest gap on a single wavelength ring with a total of $\eta - k$ nodes and $\ell - 2$ multicast destinations (and one source node) has $m$ hops. This event has probability $q_{\ell-2,\eta-k}(m)$.

For $m = k$ we obtain

$$r_{\ell-1,\eta}(k,k) = q_{\ell-1,\eta}(k) - \sum_{m=1}^{k-1} r_{\ell-1,\eta}(k,m). \tag{10}$$

To see this note that $q_{\ell-1,\eta}(k)$ represents the probability for the event that the largest spacing has $k\Lambda$ hops, whereas $\sum_{m=1}^{k-1} r_{\ell-1,\eta}(k,m)$ represents the probability for the event that the largest spacing has $k\Lambda$ hops *and* the second largest spacing has strictly less than $k\Lambda$ hops. The difference of the two probabilities thus represents the probability for the event that the largest spacing has $k\Lambda$ hops *and* the second largest spacing as $k\Lambda$ hops.

*4) Evaluation of Distribution of Length of Largest Gap:* Now we select independently and uniformly randomly the sending node of the considered multicast from among the $\eta$ nodes on wavelength $\Lambda$. The sender subdivides one of the spacings $S_1, \ldots, S_\ell$ into two subdivisions. Let $G_1, \ldots, G_{\ell+1}$ be random variables denoting the lengths of the gaps formed by placing the sender. Note that $\ell - 1$ of the formed gaps are identical to the corresponding spacings. The additional two gaps add up to the subdivided spacing. Let $G_{\max}$ be a random variable denoting the length of the largest gap, i.e., $G_{\max} = \max\{G_1, \ldots, G_{\ell+1}\}$.

For a given wavelength $\lambda \neq \Lambda$ and number $\ell$ of multicast destinations on this wavelength, we denote the distribution of the length of the largest gap by

$$c_\gamma := P(G_{\max} = \gamma), \quad \gamma = 1, \ldots, N - (\ell - 1)\Lambda. \tag{11}$$

This distribution is used in turn to evaluate the expected value of the largest gap

$$g_{\ell,\eta}^{(\lambda)} = \sum_{\gamma=1}^{N-(\ell-1)\Lambda} \gamma c_\gamma. \tag{12}$$

Note that all these quantities depend on the considered wavelength $\lambda$, the number $\ell$ of multicast destinations on the wavelength, and the total number of nodes $\eta = N/\Lambda$ on the wavelength. For the evaluation of the distribution of the length of the largest gap $c_\gamma = P(G_{\max} = \gamma),\ \gamma = 1, \ldots, N - (\ell - 1)\Lambda$, we distinguish two scenarios: (A) $\gamma$ is an integer multiple of $\Lambda$, and (B) $\gamma$ is not an integer multiple of $\Lambda$.

*a) Scenario (A): $\gamma$ is integer multiple of $\Lambda$:* In scenario (A), there are exactly three distinct cases that result in the length of the largest gap of $G_{\max} = \gamma$, with $\gamma$ being an integer multiple of $\Lambda$, i.e., $\gamma = j\Lambda,\ j \in \mathbb{N}$.

case $(i)$ Both the largest and the second largest spacing are equal to $\gamma = j\Lambda$, i.e., $S_{\max,1} = S_{\max,2} = \gamma$, which occurs with probability $r_{\ell-1,\eta}(j,j)$. In this case, even when the selected source node subdivides the largest spacing, then the largest gap still has $\gamma$ hops.

case $(ii)$ The largest spacing has $\gamma = j\Lambda$ hops ($S_{\max,1} = \gamma$), the second largest spacing has less than $\gamma$ hops ($S_{\max,2} < \gamma$), and the sender does *not* fall into the largest spacing. This event occurs with probability

$$\frac{\eta - j}{\eta} \sum_{k=1}^{j-1} r_{\ell-1,\eta}(j,k). \tag{13}$$

To see this note that the sender falls with probability $1/\eta$ on any of the $\eta$ nodes on wavelength $\Lambda$. In order not to subdivide the largest spacing, the source node is not allowed to fall onto those $j$ nodes on wavelength $\Lambda$ that are located on the ring perimeter covered by the largest spacing. This leaves $\eta - j$ nodes on which the sender may fall.

case $(iii)$ The largest spacing has more than $\gamma = j\Lambda$ hops ($S_{\max,1} > \gamma$), the second largest spacing has $\gamma$ hops ($S_{\max,2} = \gamma$), and the sender subdivides the largest spacing such that both subdivisions are shorter than $\gamma$ hops. This occurs with probability

$$\sum_{k=j+1}^{\min(\eta,2j-1)} \left( r_{\ell-1,\eta}(k,j) \cdot \frac{2j-k}{\eta} \right). \tag{14}$$

To see this consider subdividing a spacing with $k\Lambda$ hops into two subdivisions. To ensure that each subdivision is shorter than $j\Lambda$ hops, the point of subdivision can be no more than a hop distance $j\Lambda$—or equivalently a hop distance of $(k-j)\Lambda$—away from either end of the spacing with $k\Lambda$ hops. This leaves an interval of $[k - 2(k-j)]\Lambda = (2j-k)\Lambda$ hops in the middle of the spacing with $k\Lambda$ hops for the placement of the subdivision point (multicast source node). The source node falls on any of the $(2j-k)$ positions (on wavelength $\Lambda$) covered by the interval in the middle of the largest spacing with probability $1/\eta$, resulting in the probability $(2j-k)/\eta$ for the placement of the sender such that the subdivisions are shorter than $j\Lambda$ hops. In particular, if the largest spacing has $k = (2j-1)\Lambda$ hops, then the source node can only be in one position and creates the subdivisions with lengths $(j-1)\Lambda + \lambda$ hops and $j\Lambda - \lambda$ hops.

Combining all three cases we obtain for scenario (A):

$$c_{j\Lambda} = r_{\ell-1,\eta}(j,j) + \frac{\eta - j}{\eta} \sum_{k=1}^{j-1} r_{\ell-1,\eta}(j,k) + \frac{1}{\eta} \sum_{k=j+1}^{\min(\eta,2j-1)} \left\{ r_{\ell-1,\eta}(k,j) \cdot (2j-k) \right\}, \quad j = 1, 2, \ldots, \eta - (\ell - 1). \tag{15}$$

*b) Scenario (B): $\gamma$ is not an integer multiple of $\Lambda$:* Due to the rotational offset of the nodes on wavelength $\lambda$, $\lambda \neq \Lambda$ (on which we consider the multicast destinations) and wavelength $\Lambda$ (on which we consider the sender to be), the feasible values for $\gamma$ in scenario (B) are either $j\Lambda + \lambda$ or $(j+1)\Lambda - \lambda$ for some $j = 0, \ldots, \eta - 1$. The largest gap has $\gamma$ hops exactly when the largest spacing has more than $\gamma$ hops ($S_{\max,1} > \gamma$), the second largest spacing has less than $\gamma$ hops ($S_{\max,2} < \gamma$), and the sender subdivides the largest spacing such that the larger of the two subdivisions has $\gamma$ hops.

Depending on the relative rotational offset of the multicast destination nodes on $\lambda$ and the source node on $\Lambda$ there are three different cases to consider.

1. $\lambda < \Lambda/2$: In this case there is exactly one position for the sender to achieve the subdivision such that the larger subdivision has $j\Lambda + \lambda$ hops or $(j+1)\Lambda - \lambda$ hops. The sender falls on this one position with probability $1/\eta$. Hence,

$$c_{j\Lambda+\lambda} = \frac{1}{\eta} \sum_{k=j+1}^{\min(\eta-\ell+1,\ 2j)} \sum_{m=1}^{j} r_{\ell-1,\eta}(k,m), \quad j = 0, 1, \ldots, \eta - \ell \tag{16}$$

and

$$c_{(j+1)\Lambda-\lambda} = \frac{1}{\eta} \sum_{k=j+1}^{\min(\eta-\ell+1,\ 2j+1)} \sum_{m=1}^{j} r_{\ell-1,\eta}(k,m), \quad j = 0, 1, \ldots, \eta - \ell. \tag{17}$$

2. $\lambda > \Lambda/2$: This case is similar to the preceding case in that there is exactly one position for the sender to achieve the desired subdivision. Hence,

$$c_{j\Lambda+\lambda} = \frac{1}{\eta} \sum_{k=j+1}^{\min(\eta-\ell+1,\ 2j+1)} \sum_{m=1}^{j} r_{\ell-1,\eta}(k,m), \quad j = 0, 1, \ldots, \eta - \ell \tag{18}$$

and

$$c_{(j+1)\Lambda-\lambda} = \frac{1}{\eta} \sum_{k=j+1}^{\min(\eta-\ell+1,\ 2j)} \sum_{m=1}^{j} r_{\ell-1,\eta}(k,m), \quad j = 0, 1, \ldots, \eta - \ell. \tag{19}$$

3. $\lambda = \Lambda/2$: In this case there are two possible positions to achieve a subdivision such that the larger subdivision has $\gamma = j\Lambda + \Lambda/2$ hops, provided the largest spacing has at most $2j\Lambda$ hops. For instance, with a largest spacing of $2j\Lambda$ hops, the source node can be placed at a hop distance of $j\Lambda + \Lambda/2$ from either endpoint of the largest spacing. On the other hand, if the largest spacing has $(2j+1)\Lambda$ hops, then there is only one possibility to subdivide the spacing so as to achieve a largest gap of $\gamma = j\Lambda + \Lambda/2$ hops, namely by placing the source point exactly in the middle of the largest spacing. Thus,

$$c_{j\Lambda+\Lambda/2} = \frac{2}{\eta} \sum_{k=j+1}^{2j} \sum_{m=1}^{j} r_{\ell-1,\eta}(k,m) + \frac{1}{\eta} \sum_{m=1}^{j} r_{\ell-1,\eta}(2j+1,m), \qquad j = 0, 1, \ldots, \eta - \ell. \tag{20}$$

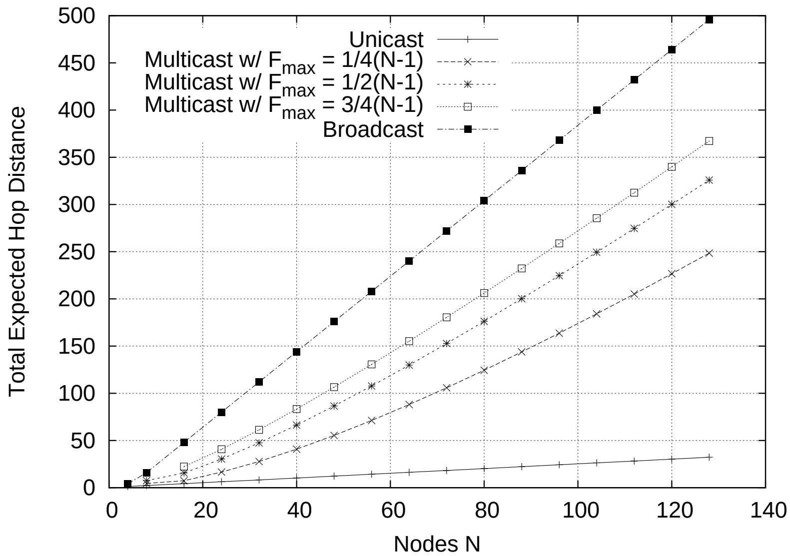


Fig. 1. Total expected hop distance $E[H]$ of shortest path routing bidirectional WDM ring with $\Lambda = 4$ vs. number of nodes $N$ for unicast, broadcast, and multicast traffic with different maximum multicast fanout $F_{\max} \in \{1/4(N-1), 1/2(N-1), 3/4(N-1)\}$.

For the special case of $\ell = 1$ multicast destination on a wavelength channel we obtain

$$g_{1,\eta}^{(\lambda)} = \begin{cases} \frac{3}{4}N & \text{for } \lambda \neq \Lambda,\ \eta \text{ even} \\ \frac{3}{4}N - \frac{\lambda}{\eta} + \frac{\Lambda}{4\eta} & \text{for } \lambda \neq \Lambda,\ \eta \text{ odd}, \lambda \leq \Lambda/2 \\ \frac{3}{4}N + \frac{\lambda}{\eta} - \frac{3\Lambda}{4\eta} & \text{for } \lambda \neq \Lambda,\ \eta \text{ odd}, \lambda \geq \Lambda/2 \\ \frac{3}{4}N - \frac{\Lambda}{4} & \text{for } \lambda = \Lambda,\ \eta \text{ odd} \\ \frac{3}{4}N - \frac{N}{4(\eta-1)} & \text{for } \lambda = \Lambda,\ \eta \text{ even}, \end{cases} \tag{21}$$

as detailed in the Appendix.

## IV. Results

Fig. 1 depicts the total expected hop distance $E[H]$ of a shortest path routing bidirectional WDM ring deploying $\Lambda = 4$ wavelengths in each direction vs. the number of nodes $N$. We consider uniform unicast, broadcast, and multicast traffic. Clearly, for unicast traffic we set the distribution of the fanout $F$ to $\mu_1 = 1$ and $\mu_l = 0$, $2 \leq l \leq N-1$. For broadcast traffic, we set $\mu_{N-1} = 1$ and $\mu_l = 0$, $1 \leq l \leq N-2$. For multicast traffic, we consider three different maximum multicast fanouts $F_{\max} \in \{1/4(N-1), 1/2(N-1), 3/4(N-1)\}$, where the fanout of a multicast packet is uniformly distributed over the interval $[2, F_{\max}]$. Apparently, for unicast and broadcast traffic we obtain the minimum and maximum $E[H]$, respectively. For multicast traffic, $E[H]$ increases for increasing $F_{\max}$. For all types of traffic, $E[H]$ increases for increasing $N$.

## V. Conclusions

The presented analytical model allows the computation of the mean hop distance of shortest path routing bidirectional WDM ring networks for a wide range of unicast, multicast, and broadcast traffic scenarios. The model is important to study WDM upgrades of currently single-channel optical rings, e.g., RPR.

*Appendix: Evaluation of Expected Length of Largest Gap $g_{1,\eta}^{(\lambda)}$.*

In this appendix we calculate the expected length of the largest gap when there is $\ell = 1$ multicast destination on the considered wavelength. We consider the five cases summarized in (21).

Case 1: $\lambda \neq \Lambda$, $\eta$ even:

$$g_{1,\eta}^{(\lambda)} = \frac{1}{\eta}\left\{\sum_{j=0}^{\frac{\eta}{2}-1}(N-(\lambda+j\Lambda)) + \sum_{j=\frac{\eta}{2}}^{\eta-1}(\lambda+j\Lambda)\right\} \tag{22}$$

$$= \frac{N}{2} - \frac{\Lambda}{\eta}\frac{\left(\frac{\eta}{2}-1\right)\frac{\eta}{2}}{2} + \frac{\Lambda}{\eta}\left(\frac{(\eta-1)\eta}{2} - \frac{\left(\frac{\eta}{2}-1\right)\frac{\eta}{2}}{2}\right) \tag{23}$$

$$= \frac{3}{4}N. \tag{24}$$

Case 2: $\lambda \neq \Lambda$, $\eta$ odd, $\lambda \leq \frac{\Lambda}{2}$:

$$g_{1,\eta}^{(\lambda)} = \frac{1}{\eta}\left\{\sum_{j=0}^{\frac{\eta-1}{2}}(N-(\lambda+j\Lambda)) + \sum_{j=\frac{\eta+1}{2}}^{\eta-1}(\lambda+j\Lambda)\right\} \tag{25}$$

$$= \frac{N}{2}\frac{\eta+1}{\eta} - \frac{\lambda}{\eta} + \frac{\Lambda}{\eta}\left(\frac{(\eta-1)\eta}{2} - \frac{(\eta-1)(\eta+1)}{4}\right) \tag{26}$$

$$= \frac{3}{4}N - \frac{\lambda}{\eta} + \frac{\Lambda}{4\eta}. \tag{27}$$

Case 3: $\lambda \neq \Lambda$, $\eta$ odd, $\lambda \geq \frac{\Lambda}{2}$: This case can be mapped into Case 2 by mapping $\lambda \to \Lambda - \lambda$, that is, we obtain the expected hop distance for a wavelength $\lambda$ falling into Case 3 by plugging $\Lambda - \lambda$ into (27):

$$g_{1,\eta}^{(\lambda)} = \frac{3}{4}N - \frac{\Lambda-\lambda}{\eta} + \frac{\Lambda}{4\eta} \tag{28}$$

$$= \frac{3}{4}N + \frac{\lambda}{\eta} - \frac{3\Lambda}{4\eta}. \tag{29}$$

Case 4: $\lambda = \Lambda$: With $h(1,\eta)$ denoting the expected hop distance required to reach one destination on a single-wavelength ring with $\eta$ nodes, we have

$$g_{1,\eta}^{(\Lambda)} = \Lambda \cdot (\eta - h(1,\eta)). \tag{30}$$

Noting that

$$h(1,\eta) = \begin{cases} \frac{\eta+1}{4} & \text{for } \eta \text{ odd} \\ \frac{\eta^2}{4(\eta-1)} & \text{for } \eta \text{ even} \end{cases} \tag{31}$$

the result in (21) follows.

## REFERENCES

[1] M. Maier, M. Scheutzow, M. Herzog, and M. Reisslein, "Multicasting in IEEE 802.17 Resilient Packet Ring (RPR)," *IEEE Transactions on Communications, submitted*, 2005.

[2] M. Scheutzow, P. Seeling, M. Maier, and M. Reisslein, "Multicast Capacity of Packet-Switched Ring WDM Networks," in *Proc., IEEE INFOCOM*, Miami, FL, Mar. 2005, pp. 706–717.