

# An Introduction to Kalman Filtering with MATLAB Examples

Narayan Kovvali, Mahesh Banavar, and Andreas Spanias  
SenSIP Center, Arizona State University

*SYNTHESIS LECTURES ON SIGNAL PROCESSING #12*



MORGAN & CLAYPOOL PUBLISHERS

## ABSTRACT

The Kalman filter is the Bayesian optimum solution to the problem of sequentially estimating the states of a dynamical system in which the state evolution and measurement processes are both linear and Gaussian. Given the ubiquity of such systems, the Kalman filter finds use in a variety of applications, e.g., target tracking, guidance and navigation, and communications systems. The purpose of this book is to present a brief introduction to Kalman filtering. The theoretical framework of the Kalman filter is first presented, followed by examples showing its use in practical applications. Extensions of the method to nonlinear problems and distributed applications are discussed. A software implementation of the algorithm in the MATLAB programming language is provided, as well as MATLAB code for several example applications discussed in the manuscript.

## KEYWORDS

dynamical system, parameter estimation, tracking, state space model, sequential Bayesian estimation, linearity, Gaussian noise, Kalman filter

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